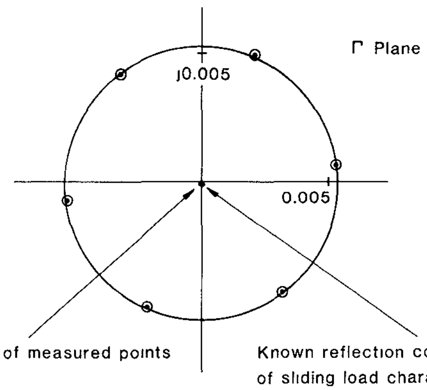

Fig. 3. Measured reflection coefficients of a sliding load at 6 equispaced positions The center of the best-fit circle is within 0.00007 of the known reflection coefficient of the characteristic impedance of the sliding load

which not only ensures at least 20-dB additional isolation between diodes, but also stabilizes their impedances as seen from the six-port junction. Some diode types produce at their microwave port a fraction of the detected dc voltage, and therefore each is also preceeded by a dc block on the inner coaxial conductor to eliminate dc cross-talk.

To illustrate the viability of the operating mode, a six-port [3] was calibrated using five standard terminations [6], and Fig. 3 shows the results of measurements made at 5 GHz on a sliding load (with a residual VSWR of approximately 1.01) moved in equal increments. The standard deviation of the measured values of $|\Gamma|$ from the best-fit circle was less than 0.00003, and the magnitude of the difference between the mean of the measurements and the known value of the reflection coefficient of the characteristic impedance of the sliding load was less than 0.00007. The standard deviation of the difference between the measured arguments of Γ was less than 0.85 degrees, which is noteworthy considering that angular definition is indeterminate at the origin.

A test at 5 GHz was made on a sliding short-circuit moved in equal increments. The standard deviation of the differences between the measured arguments of Γ was less than 0.08 degrees, corresponding to a physical displacement of 0.0066 mm.

These results show that six-port operation using uncalibrated nonlinear diodes is no less accurate than methods using calibrated diodes [3], [5], [6].

A variation of the operating mode described above is to replace one of the diodes (say diode 4) with a linear power meter, and use the remaining three diodes in turn as leveling loop detectors, thus providing three values of L_i, $i=1$, 2, and 3. The value of L_4 is a constant, K, as can be seen by imagining a fourth diode to be in parallel with the power meter, and used as a leveling detector. Thus

$$P_i/P_4 = K/L_i, \qquad i=1,2,3.$$

with K assigned an arbitrary positive value. Since only three diodes are used, there is a 25-percent reduction in measurement time, but the method is sensitive to changes in the leveling reference voltage (effectively, changes in K), whereas the four-diode mode is largely immune to leveling reference changes, with any sensitivity being solely due to differences in diode characteristics. Such differences are small at the low diode operating level.

IV. Conclusions

Multiplexing four diodes to act in turn as sensors for a closed loop leveling circuit, and recording the corresponding power readings of a single linear power meter, eliminates the need for either a) four linear power meters, and therefore in practice, the use of relatively high powers, or b) the calibration of semiconductor diodes to be used as low-level power meters.

By operating all the diodes at a fixed level, sensitivity changes (with temperature of time) may be easily accomodated by the use of scaling factors.

The operating mode also offers flexibility in the choice of diodes, since their linearity is not a consideration. Instead, low $1/f$ noise may be a selection criterion, suggesting, for instance, the use of tunnel diodes.

The only disadvantage of the mode of operation is that nonlinear (level dependent) impedances cannot be measured, since the level at the measuring port is not held constant during the measurement.

References

[1] G. F. Engen, "The six-port reflectometer: An alternative network analyzer," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 1075–1083, Dec 1977

[2] H. M. Cronson and L. Susman, "A dual six-port network analyzer," *IEEE Trans. Microwave Thorey Tech.*, vol MTT-29, pp. 372–378, Apr 1981.

[3] P. I. Somlo and J. D. Hunter, "A six-port reflectometer and its complete characterization by convenient calibration procedures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 186–192, Feb. 1982

[4] J. R. Juroshek and C. A. Hoer, "A dual six-port network analyzer using diode detectors," *IEEE Trans Microwave Theory Tech.*, vol MTT-32, pp. 78–82, Jan. 1984.

[5] P I. Somlo, "The case for using a matched load standard for six-port calibration," *Electron. Lett*, vol 19, pp 979–980, Nov. 1983.

[6] J. D. Hunter and P. I. Somlo, "An explicit six-port calibration method using five standards," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp 69–72, Jan 1985.

Scalar Variational Analysis of Single-Mode Waveguides with Rectangular Cross Section

PRASANNA K. MISHRA, ANURAG SHARMA, S. LABROO, AND A. K. GHATAK

Abstract —**We use the variational method to analyze single-mode optical waveguides with rectangular cross section. In particular, we propose a new trial field and show that it gives much better results and involves less computational effort as compared to other trial functions.**

I. Introduction

Single-mode optical waveguides with rectangular cross sections are the building blocks of most of the devices in integrated optics, and, hence, a knowledge of their propagation characteristics is important for the design of such devices. However, it is not possible to solve the electromagnetic boundary value problem analytically to obtain the propagation characteristics of such

Manuscript received July 31, 1984; revised October 16, 1984. This work was supported in part by the Electronics Commission, Government of India.

The authors are with the Department of Physics, Indian Institute of Technology, New Delhi-110016, India. P K. Mishra is currently on study leave from G. M. College, Sambalpur-768004, Orissa, India

0018-9480/85/0300-0282$01.00

waveguides, and one has to use numerical or approximate techniques. The numerical techniques, such as circular harmonic analysis [1], finite-element method [2], field expansion to orthogonal functions [3], [4], etc., involve extensive computation and do not lead to simple analytical forms for the modal fields. Hence, various approximate methods have been developed, the two main methods being the one developed by Marcatili [5], and the effective index method [6]. However, both these methods are accurate only for modes far from cutoff and lead to considerable error in approximating the fundamental mode in the region of interest for single-mode operation. It has been shown that [7]–[9] the variational method yields more accurate results in comparison to other approximate methods if a judicious choice of the trial field, approximating the modal field of single-mode waveguide, is made. The objective of the present paper is to consider appropriate trial fields and compare their suitability for approximating the propagation characteristics of single-mode optical waveguides with rectangular cross section. We restrict our studies to the scalar approximation which has been used extensively in the past [2], [3], [7]–[11] and holds good for most practical waveguides in which the index difference is small.

II. Analysis

We consider a general waveguide with a refractive index distribution given by

$$\begin{aligned} n^2(x,y) &= n_0^2, && |x|<b, |y|<T \\ &= n_p^2, && |x|>b, |y|<T \\ &= n_c^2, && y>T \\ &= n_s^2, && y<-T. \end{aligned} \tag{1}$$

Such a configuration reduces to a rectangular waveguide when $n_p = n_c = n_s < n_0$, to a channel waveguide when $n_c < n_p = n_s < n_0$, and to an embossed waveguide when $n_c = n_p < n_s < n_0$.

The scalar variational expression for the fundamental mode propagation constant β is given by [10]

$$\beta^2 = \frac{k_0^2 \iint_{-\infty}^{\infty} n^2(x,y)|\psi_t|^2\,dx\,dy - \iint_{-\infty}^{\infty} |\nabla_t \psi_t|^2\,dx\,dy}{\iint_{-\infty}^{\infty} |\psi_t|^2\,dx\,dy} \tag{2}$$

where k_0 is the free-space wavenumber, $\nabla_t = \nabla - \partial/\partial z$, $n^2(x,y)$ is defined by (1), and $\psi_t(x,y)$ is the trial field containing a certain number of parameters with respect to which β^2 is maximized. In terms of dimensionless parameters, (2) can be written as

$$U^2 = \frac{b^2\left[k_0^2 \iint_{-\infty}^{\infty} \{n_0^2 - n^2(x,y)\}|\psi_t|^2\,dx\,dy + \iint_{-\infty}^{\infty}\left\{\left|\frac{d\psi_t}{dx}\right|^2 + \left|\frac{d\psi_t}{dy}\right|^2\right\}dx\,dy\right]}{\iint_{-\infty}^{\infty} |\psi_t|^2\,dx\,dy} \tag{3}$$

where

$$U^2 = b^2\left(k_0^2 n_0^2 - \beta^2\right).$$

III. Variational Trial Fields

Trial fields for the fundamental mode can, in general, be constructed either by considering a finite expansion in terms of some appropriate mutually orthogonal functions with the expansion coefficients as variational parameters [11], [12], or by an appropriate single function involving the variational parameters [7]–[9]. The former approach is useful for analyzing multimode structures since it can give propagation constants for a large number of the guided modes simultaneously. Further, the trial field involves linear combinations of many functions and it may not be convenient for obtaining field-related characteristics of the waveguide, such as source to waveguide coupling efficiency. On the other hand, a single function trial field has the advantage that, once the parameters are known, the approximation of the field is in a very simple closed form. In the following, we discuss some of the useful single function trial fields and compare their accuracy. The trial fields are assumed to be separable in the X and Y directions, i.e.,

$$\psi_t(x,y) = \psi_x(x)\cdot\psi_y(y). \tag{4}$$

The specific forms for ψ_x and ψ_y are given below.

A. *Double-Gaussian (DG) Trial Field*

The trial field is assumed to be Gaussian, both inside and outside the core in the X and Y directions. This is one of the simplest trial fields and can be written as

$$\begin{aligned} \psi_x(x) &= \exp[-\alpha x^2] \\ \psi_y(y) &= \exp\left[-\eta_+(y-d)^2\right], && y>d \\ &= \exp\left[-\eta_-(y-d)^2\right], && y<d \end{aligned} \tag{5}$$

where $\eta_+ \neq \eta_-$ and $d \neq 0$ for channel and embossed waveguides. Substituting these ψ_x and ψ_y in (3) and (4), one obtains an expression for U^2 as a function of the variational parameters α, η_+, η_-, and d. All integrations can be carried out analytically with some resulting in error functions (see Appendix). The value of U^2 is minimized by varying the four parameters using a standard minimization routine. The minimum value of U^2 obtained in this way would approximate the exact U^2 value, and the corresponding values of α, η_+, η_-, and d when substituted in (5) gives an approximation to the modal field.

B. *Gaussian-Exponential (GE) Trial Field*

In this case, the trial field is chosen to be Gaussian inside the core and an exponentially decaying function outside the core, i.e.,

$$\begin{aligned} \psi_x(x) &= \exp[-sx^2], && |x|\leqslant b \\ &= \exp[-sb(2x-b)], && x>b \\ &= \exp[sb(2x+b)], && x<-b \\ \psi_y(y) &= \exp[-t_1(y-c)^2], && T>y>c \\ &= \exp[-t_1(T-c)(2y-T-c)], && y>T \\ &= \exp[-t_2(y-c)^2], && -T<y<c \\ &= \exp[t_2(T+c)(2y+T-c)], && y<-T. \end{aligned} \tag{6}$$

The form of this field is closer to the actual modal field, since the field sufficiently away from the core indeed varies as an exponential and not as a tail of a Gaussian as in the DG-field. When substituted in (3) and (4), this trial field again leads to an analytical expression for U^2 as a function of s, t_1, t_2, and c. This expression also involves error functions (see Appendix) and requires a four parameter minimization. Thus, the computational effort involved is almost the same as that with DG-trial field.

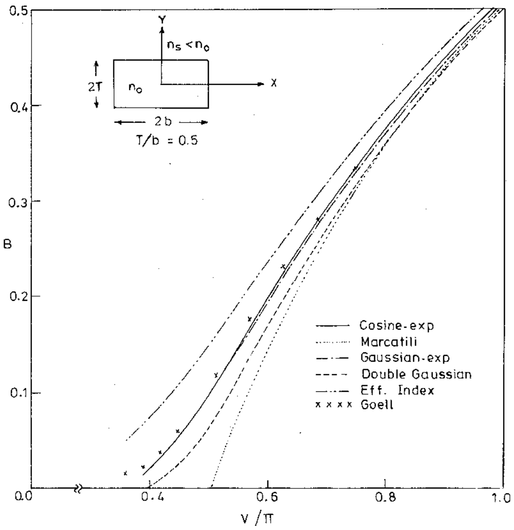

Fig. 1. Normalized propagation constant B as a function of normalized frequency V for a rectangular waveguide. The exact results of Goell [1] are transferred directly from his Fig. 17.

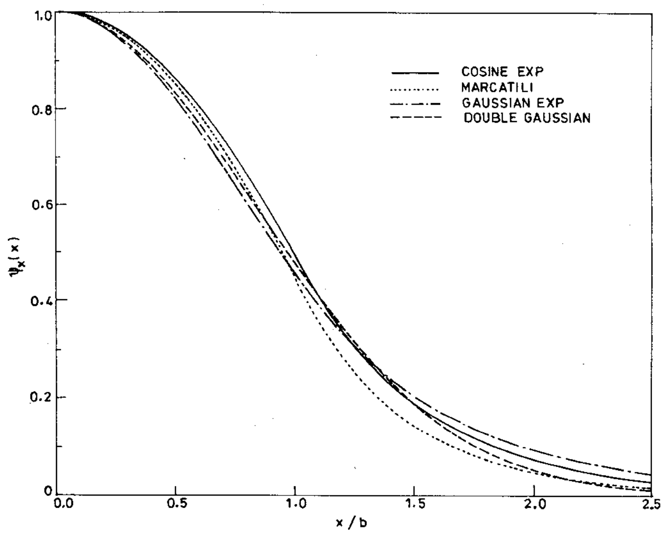

Fig. 2. Variation of modal field versus normalized distance x/b along the x-direction for the rectangular waveguide of Fig. 1 for $V = 0.8\ \pi$.

C. Cosine-Exponential (CE) Trial Field

Here we choose the trial field to be a cosine function inside the core and an exponentially decaying function outside the core. In fact, the corresponding trial fields ψ_x and ψ_y have the same form as the exact mode for a step index slab waveguide. This trial field can be written as

$$\begin{aligned}
\psi_x(x) &= \cos px, & |x| \leqslant b \\
&= \cos pb \cdot \exp[-p \tan pb \cdot (|x| - b)], & |x| \geqslant b \\
\psi_y(y) &= \cos(qT - \sigma)\exp[-q \tan(qT - \sigma)(y - T)], & y > T \\
&= \cos(qy - \sigma), & |y| \leqslant T \\
&= \cos(qT + \sigma)\exp[q \tan(qT + \sigma)(y + T)], & y < -T.
\end{aligned} \tag{7}$$

The trial field corresponds to the fundamental mode of a guiding structure consisting of two mutually perpendicular slab-waveguides, the refractive index distribution which can be obtained from the values of the parameters $p \cdot q$ and σ. The above trial field leads to an analytical expression for U^2 in terms of $p \cdot q$ and σ involving only simple trigonometric functions (see Appendix). Further, it involves only three parameters, and, therefore, the computational effort is much less than with the GE or DG trial fields.

IV. Comparison of Trial Fields

In this section, we compare the accuracy and suitability of the trial fields discussed above with the help of some typical examples. We also make a comparison with Marcatili's method [5] and the effective index method [6]. First, we consider a rectangular waveguide ($n_p = n_c = n_s$) with $n_0 - n_s \ll n_0$ having an aspect ratio $T/b = 0.5$. The variation of the normalized propagation constant $B(= 1 - U^2/V^2)$ with the normalized frequency $V/\pi = 2b/\lambda(n_0^2 - n_s^2)^{1/2}$ is shown in Fig. 1 for different trial fields along with the result obtained by using Marcatili's analysis and the effective index method. Since the variational analysis would never result in a higher value of B than its exact value, we can safely conclude that the trial field which gives the highest value of B is the most accurate field.[1] Thus, the figure shows that the CE trial field gives the best results even at low V values. In fact, in the region of interest for single-mode operation, i.e., for normalized frequencies around and below $V \approx 0.85\ \pi$ (the cutoff frequency of the next higher mode [1]), the curve corresponding to the CE trial field almost coincides with the results of the numerical method of Goell [1]. The DG trial field is the least accurate of the three trial fields considered here. However, it is still more accurate than Marcatili's method. The GE field is only slightly inaccurate as compared to the CE field, but the latter requires much less computational effort. Fig. 2, where the field is plotted along the $y = 0$ axis for a rectangular waveguide with $V/\pi = 0.8$, shows that the field corresponding to Marcatili's method, and the DG and GE trial fields, are considerably different from the CE trial field and, hence, are in considerable error.

Next we consider asymmetric waveguides. In Fig. 3, we have plotted the dispersion curves for a channel waveguide ($n_p = n_s$) and in Fig. 4, for an embossed waveguide ($n_p = n_c$) with $n_0 - n_s \ll n_0$, $T/b = 0.5$, and $n_c = n_0/1.5$. These figures show that even for such waveguides, the CE trial field gives much better results in comparison to other methods. Further, it may be noted that, although the GE trial field is an accurate approximation for single-mode diffused channel waveguide [9], it is quite inaccurate for step index waveguides, especially for asymmetric waveguides. The modal fields along the y-direction for a channel waveguide with $V/\pi = 0.8$ are plotted in Fig. 5 along with the field obtained using Marcatili's method, which is essentially the modal field of an asymmetric slab waveguide obtained by ignoring the X-variation of the refractive index.

V. Summary and Conclusion

In this paper, we have analyzed optical waveguides with rectangular cross section using single function trial fields in the variational formulation. In particular, we have considered the double Gaussian [13] and the Gaussian-exponential [9] trial fields which have been used in the past to analyze diffused waveguides. In addition, we have proposed a new trial field, the cosine-exponential trial field, and have compared the performance of this field. Our calculations show that the cosine-exponential field requires much less computational effort and gives much more accurate

[1] This is strictly true as long as the trial fields are such that they satisfy the same boundary and continuity conditions as the exact scalar modal field does, i.e., the field and its first derivative are continuous everywhere and vanish at large (infinite) distances away from the waveguide. This is true for all the trial fields considered in the present study. A general discussion on the variational method and its applicability is given by T. K. Sarkar, *Radio Science*, vol. 18, pp. 1207–1224, Dec. 1983.

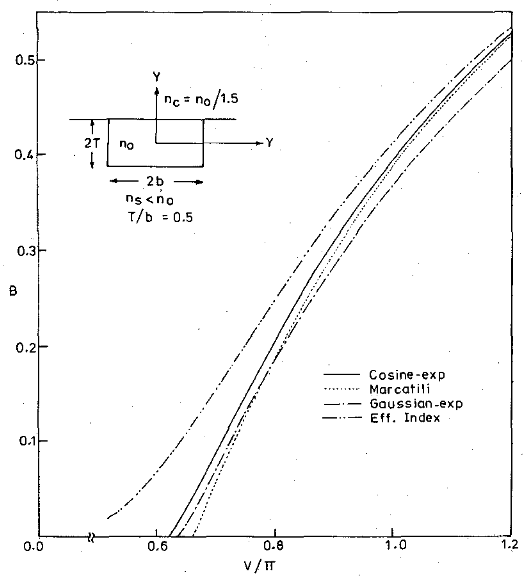

Fig. 3. Normalized propagation constant B as a function of normalized frequency V for a channel waveguide.

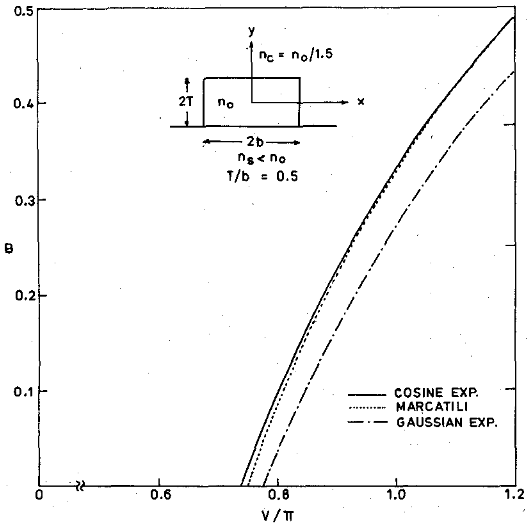

Fig. 4. Normalized propagation constant B as a function of normalized frequency V for an embossed waveguide.

results in comparison to the DG and GE fields. This also holds for channel as well as embossed waveguides throughout the single-mode region. Further, it is much more accurate than the commonly used effective index method [6] and Marcatili's method [5].

An additional advantage with the cosine-exponential field is that one can obtain an equivalent guiding structure which can then be used in further studies such as obtaining vector modes, analyzing directional couplers, etc. Some of the results of such a study have already been reported [14]. Further work along these lines is in progress and will be reported elsewhere.

Appendix
Expressions for U^2 Using Different Trial Fields

A. Double Gaussian (DG)

$$U^2 = V^2\frac{\eta_+}{\eta_+ + \eta_-}(1-E_-) + V_c^2\frac{\eta_-}{\eta_+ + \eta_-}(1-E_+) + V_p^2\frac{\eta_+ E_- + \eta_- E_+}{\eta_+ + \eta_-}(1-\operatorname{erf}\alpha) + \frac{1}{2}\left(\alpha^2 + \eta_+\eta_-\right)$$

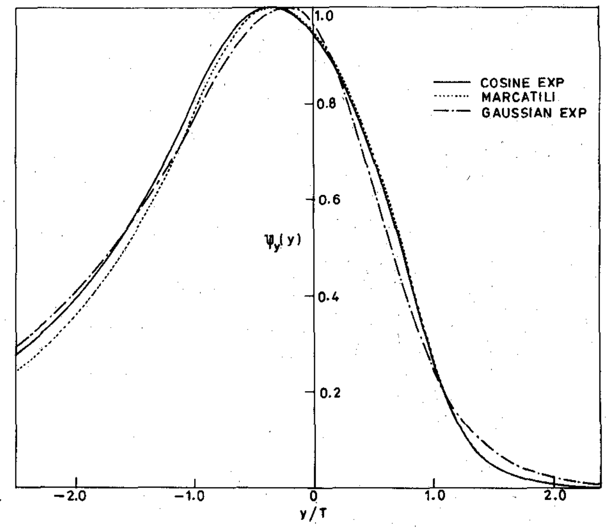

Fig. 5. Variation of modal field versus normalized distance for the channel waveguide of Fig. 3 for $V = 0.8\pi$.

where $\boldsymbol{\alpha} = \sqrt{2\alpha}\, b$, $\boldsymbol{\eta}_\pm = \sqrt{2\eta_\pm}\, b$, $E_\pm = \operatorname{erf}[\boldsymbol{\eta}_\pm(\boldsymbol{T} \mp \boldsymbol{d})]$, $\boldsymbol{T} = T/b$, $\boldsymbol{d} = d/b$, $V^2 = k_0^2 b^2(n_0^2 - n_s^2)$, and $V_{c,p}^2 = k_0^2 b^2(n_0^2 - n_{c,p}^2)$.

B. Gaussian-Exponential (GE)

$$U^2 = \frac{1}{IJ}\left[V^2 IJ_1 + V_c^2 IJ_4 + 2V_p^2 I_2(J_2 + J_3) + \frac{1}{2}\left\{s^2 I_1 J + I\left(t_2^2 J_2 + t_1^2 J_3\right)\right\}\right]$$

where

$$I_1 = \pi^{1/2}\operatorname{erf}(\boldsymbol{s})/\boldsymbol{s}$$
$$I_2 = \frac{1}{2}\exp(-\boldsymbol{s}^2)/\boldsymbol{s}^2$$
$$I = I_1 + 2I_2$$
$$J_1 = \exp\left[-\boldsymbol{t}_2^2(\boldsymbol{T}+\boldsymbol{c})^2\right]/2\boldsymbol{t}_2^2(\boldsymbol{T}+\boldsymbol{c})$$
$$J_2 = \frac{\pi^{1/2}}{2\boldsymbol{t}_2}\operatorname{erf}[\boldsymbol{t}_2(\boldsymbol{T}+\boldsymbol{c})]$$
$$J_3 = \frac{\pi^{1/2}}{2\boldsymbol{t}_1}\operatorname{erf}[\boldsymbol{t}_1(\boldsymbol{T}-\boldsymbol{c})]$$
$$J_4 = \exp\left[-\boldsymbol{t}_1^2(\boldsymbol{T}-\boldsymbol{c})^2\right]/2\boldsymbol{t}_1^2(\boldsymbol{T}-\boldsymbol{c})$$
$$J = J_1 + J_2 + J_3 + J_4$$
$$\boldsymbol{c} = c/b,\ \boldsymbol{s} = \sqrt{2s}\, b,\ \boldsymbol{t}_{1,2} = \sqrt{2t_{1,2}}\, b.$$

C. Cosine-Exponential (CE)

$$U^2 = \frac{V^2 I_{y1} + V_c^2 I_{y3}}{I_y} + \frac{2V_p^2 I_{y2} I_{x1}}{I_x I_y} + \frac{\boldsymbol{p}^2}{I_x} + \frac{\boldsymbol{q}^2}{\boldsymbol{T}^2 I_y}$$

where

$$I_{x1} = \cos^3 \boldsymbol{p}/2\boldsymbol{p}\sin \boldsymbol{p}$$
$$I_{x2} = 1 + \sin 2\boldsymbol{p}/2\boldsymbol{p}, \quad I_x = 2I_{x1} + I_{x2}$$
$$I_{y1} = \cos^3(\boldsymbol{q}-\sigma)/2\boldsymbol{q}\sin(\boldsymbol{q}-\sigma)$$
$$I_{y2} = 1 + \sin 2\boldsymbol{q}\cdot\cos 2\sigma/2\boldsymbol{q}$$
$$I_{y3} = \cos^3(\boldsymbol{q}+\sigma)/2\boldsymbol{q}\sin(\boldsymbol{q}+\sigma), \quad I_y = I_{y1} + I_{y2} + I_{y3}$$
$$\boldsymbol{p} = pb \text{ and } \boldsymbol{q} = qT.$$

References

[1] J. E. Goell, "A circular-harmonic computer analysis of rectangular dielectric waveguides," *Bell Syst. Tech. J.*, vol. 48, pp. 2133–2160, Sept. 1969.

[2] C. Yeh, K. Ha, S. B. Dong, and W. P. Brown, "Single-mode optical waveguides," *Appl. Opt.*, vol. 18, pp. 1490–1504, May 1979.

[3] M. Matsuhara, "Analysis of TEM modes in dielectric waveguides by a variational method," *J. Opt. Soc. Amer.*, vol. 63, pp. 1514–1517, Dec. 1973.

[4] H. F. Taylor, "Dispersion characteristics of diffused channel waveguides," *IEEE J. Quantum Electron.*, vol. QE-12, pp. 748–752, 1976.

[5] E. A. J. Marcatili, "Dielectric rectangular waveguide and directional coupler for integrated optics," *Bell Syst. Tech. J.*, vol. 48, pp. 2071–2102, Sept 1969.

[6] G. B. Hocker and W. K. Burns, "Mode dispersion in diffused channel waveguides by the effective index method," *Appl. Opt.*, vol. 16, pp. 113–118, Jan. 1977.

[7] A. Sharma and A. K. Ghatak, "Variational analysis of single mode integrated optical waveguides," in *Proc. 8th All India Symposium on optics*, (New Delhi, India), Mar. 21–24, 1980, pp. 12.

[8] A. Sharma, E. Sharma, I. C. Goyal, and A. K. Ghatak, "Variational analysis of directional couplers with graded index profile," *Opt. Commun.*, vol. 34, pp. 39–42, July 1980.

[9] U. Jain, A. Sharma, K. Thyagarajan, and A. K. Ghatak, "Coupling characteristics of a diffused channel-waveguide directional coupler," *J. Opt. Soc. Amer.*, vol. 72, pp. 1545–1549, Nov. 1982.

[10] M. J. Adams, *An Introduction to Optical Waveguides.* Chichester: Wiley, 1981.

[11] F. P. Payne, "A new theory of rectangular optical waveguides," *Opt. Quantum Electron.*, vol. 14, pp. 525–537, 1982.

[12] K. Ogusu, "Optical strip waveguides: A detailed analysis including leaky modes," *J. Opt. Soc. Amer.*, vol. 73, pp. 353–357, Mar. 1983.

[13] L. McCaughan and E. J. Murphy, "Influence of temperature and initial titanium dimensions on fiber Ti: $LiNbO_3$ waveguide insertion loss at $\lambda = 1.3\ \mu m$," *IEEE J. Quantum Electron.*, vol. QE-19, pp. 131–135, Feb. 1983.

[14] A. Sharma, P. K. Mishra, and A. K. Ghatak, "Analysis of single mode waveguides and directional couplers with rectangular cross section," in *Proc. 2nd European Conf. on Integrated Optics*, (Italy), Oct. 17–18, 1983, pp. 9–12.

Letters

Comments on "A Rigorous Technique for Measuring the Scattering Matrix of a Multiport Device with a Two-Port Network Analyzer"

E. VAN LIL, MEMBER, IEEE

In the above paper,[1] Tippet and Speciale gave expressions for the correction to be made on the S matrix to account for the mismatches on the ports not connected to the network analyzer.

The basic transformation was given by

$$S' = \left((I-S)^{-1}(I+S) - (I+\Gamma)(I-\Gamma)^{-1}\right) \cdot \left((I-S)^{-1}(I+S) + (I+\Gamma)(I-\Gamma)^{-1}\right)^{-1} \quad (1)$$

(notations as in the above paper[1] and [1]) from which the authors derived

$$S' = (I-S)^{-1}(S-\Gamma)(I-S\Gamma)^{-1}(I-S). \quad (2)$$

By using the relation

$$(I-A)^{-1}(I+A) = (I+A)(I-A)^{-1} \quad (3)$$

that can be easily proven by multiplying each side both right and left with $(I-A)$, we can rewrite (1) as

$$S' = \left((I+S)(I-S)^{-1} - (I-\Gamma)^{-1}(I+\Gamma)\right) \cdot \left((I+S)(I-S)^{-1} + (I-\Gamma)^{-1}(I+\Gamma)\right)^{-1}. \quad (4)$$

By following the same procedure as used in the derivation of (2) from (1), we obtain

$$S' = (I-\Gamma)^{-1}\left((I-\Gamma)(I+S) - (I+\Gamma)(I-S)\right)(I-S)^{-1} \cdot (I-S)\left((I-\Gamma)(I+S) + (I+\Gamma)(I-S)\right)^{-1}(I-\Gamma)$$

or

$$S' = (I-\Gamma)^{-1}(S-\Gamma)(I-\Gamma S)^{-1}(I-\Gamma) \quad (5)$$

proving the identity of (5) and (2) as was expected by Tippet and Speciale.

The simplification of Dropkin [1] applied by Tippet and Speciale to (5) gave

$$S' = (I+\Gamma)S(I-\Gamma S)^{-1}(I-\Gamma) - \Gamma \quad (6)$$

but does not mean a significant improvement in computational efficiency, because $I-\Gamma$, Γ, $I+\Gamma$, and $(I-\Gamma)^{-1}$ are diagonal matrices. So, (6) is only a little bit more efficient than (5) because it does not involve a division by $I-\Gamma$ but rather a multiplication by $I+\Gamma$. Furthermore, if a whole series of unknown N ports has to be measured, the reflection coefficients in the diagonal matrix Γ are known, so that only the computation of $S(I-\Gamma S)^{-1}$ has to be carried out, followed by a multiplication of column i by $1-\Gamma_i$, row j by $1+\Gamma_j$ and a subtraction of Γ_k from diagonal element k. The formula by Dropkin [1], namely

$$S' = S - (I+S)\Gamma(I-S\Gamma)^{-1}(I-S) \quad (7)$$

even if it does contain a significant improvement over (2), it still is much less efficient than (6). Indeed, only the operation $(I+S)\Gamma$ or $\Gamma(I-S\Gamma)^{-1}$ can make use of the diagonal form of Γ. So, (6) gains a whole matrix multiplication and most of a matrix subtraction in computational effort over (7).

In the general case of an N-port measured with an M-port network analyzer, it is easy to show that (6) needs to be applied at most $N!/(M!(N-M)!)$ times for a $M \times M$ matrix and once

Manuscript received August 6, 1984.

The author is Research Associate of the National Fund for Scientific Research (Belgium). He is with the Catholic University of Leuven, Dept Elektrotechniek, Kardinaal Mercierlaan, 94, B-3030, Heverlee, Belgium.

[1] J. C. Tippet and R. A. Speciale, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 661–666, May 1982.

0018-9480/85/0300-0286$01.00 ©1985 IEEE